➤➤➤ POLYNOMIALS ⮜⮜⮜

ML – 5

ZEROS OR ROOTS OF A POLYNOMIAL

A real number α is a root or zero of polynomial f(x) = $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0$,

if $f(\alpha)$ = 0. i.e. $a_n\alpha^n + a_{n-1} + a_{n-2}\alpha^{n-2} + ... + a_1\alpha + a_0 = 0$.

For example x = 3 is root of the polynomial f(x) = $x^3 - 6x^2 + 11x - 6$, because

f(3) = $(3)^3 - 6(3)^2 + 11(3)$ - = 27 - 54 + 33 - 6 = 0.

but x = - is not a root of the above polynomial,

$\because$ f(-2) = (-2).3 - 6(-2)^2 + 11(-2) - 6

f(-2) = -8 -24 - 22 - 6

f(-2) = - 60 $\neq$ 0.

(a) Value of a Polynomial :

The value of a polynomial f(x) at x= α is obtained by substituting x = α in the given polynomial and is denoted by $f(\alpha)$. e.g. If f(x) = $2x^3 - 13x^2 + 17x + 12$ then its value at x = 1 is.

f(1) = $2(1)^3 - 13(1)^2 + 17(1) + 12$

= 2 - 13 + 17 + 12 = 18.

Ex.1 Show that x = 2 is a root of $2x^3 + x^2 - 7x - 6$.

Sol. p(x) = $2x^3 + x^2 - 7x - 6$ then,

p(2) = $2(2)^3 + (2)^2 7(2) - 6$ = 16 + 4 - 14 - 6 = 0

Hence x = 2 is a root of p(x). **Ans.**

Ex. 2 If $x = \frac{4}{3}$ is a root of the polynomial f(x) = $6x^3 - 11x^2 + kx - 20$ then find the value of k.

Sol. f(x) = $6x^3 - 11x^2 + kx - 20$

$\Rightarrow f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$

$\Rightarrow 6.\frac{64}{9.3} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$

$\Rightarrow$ 128 - 176 + 12k - 180 = 0

$\Rightarrow$ 12k + 128 - 356 = 0

$\Rightarrow$ 12k = 228

$\Rightarrow$ k = 19 **Ans.**

Ex.3 If x = 2 & x = 0 are two roots of the polynomial f(x) = $2x^3 - 5x^2 + ax + b$. Find the values of and b.

Sol. $f(x) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow$ $16 - 20 + 2a + b = 0$

$\Rightarrow$ $2a + b = 4$ ....(i)

$\Rightarrow$ $f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0$

$\Rightarrow$ $b = 0$

So, $2a = 4$

Hence, $a = 2, b = 0$ **Ans.**

REMAINDER THEOREM

Let 'p(x)' be any polynomial of degree greater than or equal to one and a be any real number and If p(x) is divided by (x - a), then the remainder is equal to p(a).

Let q(x) be the quotient and r(x) be the remainder when p(x) is divided by (x - a) then

Dividend = Divisor × Quotient + Remainder

$p(x) = (x - a) \times q(x) + [r(x) \text{ or } r]$, where $r(x) = 0$ or degree of r(x) < degree of (x -). But (x - a) is a polynomial of degree 1 and a polynomial of degree less than 1 is a constant. Therefore, either $r(x) = 0$ or $r(x) =$ Constant.

Let $r(x) = r$, then $p(x) = (x - a)q(x) + r$,

putting x = a in above equation $p(a) = (a - a)q(a) + r = 0.\ q(a) + r$

$p(a) = 0 + r$

$\Rightarrow$ $p(a) = r$

This shows that the remainder is p(a) when p(x) is divided by (x - a).

REMARK : If a polynomial p(x) is divided by (x + a), (ax - b), (x + b), (b - ax) then the remainder in the value of p(x) at $x = -a, \frac{b}{a}, -\frac{b}{a}, \frac{b}{a}$ i.e. p(-a), $p\left(\frac{b}{a}\right), p\left(-\frac{b}{a}\right), p\left(\frac{b}{a}\right)$ respectively.

Ex.4 Find the remainder when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $g(x) = 1 - 2x$.

Sol. $1 - 2x = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4$$

$$= \frac{1}{8} - \frac{3}{2} + 1 - 4$$

$$= \frac{1 - 12 + 8 - 32}{8} = -\frac{35}{8}$$ **Ans.**

Ex.5 The polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ are divided by x + 2 if the remainder in each case is the same, find the value of a.

Sol. $p(x) = ax^3 + 3x^2 - 13$ and $q(x) = 2x^3 - 5x + a$

when p(x) & q(x) are divided by $x + 2 = 0 \Rightarrow x = -2$

$p(-2) = q(-2)$

$\Rightarrow a(-2)^3 + 3(-2)^2 - 13 = 2(-2)^3 - 5(-2) + a$

$\Rightarrow -8a + 12 - 13 = -16 + 10 + a$

$\Rightarrow$ $-9a = -5$

$\Rightarrow$ $a = \frac{5}{9}$ **Ans.**

(a) Factor Theorem :

Let p(x) be a polynomial of degree greater than or equal to 1 and 'a' be a real number such that p(a) = 0, than (x - a) is a factor of p(x). Conversely, if (x - a) is a factor of p(x), then p(a) = 0.

Ex.6 Show that x + 1 an d 2x - 3 are factors of $2x^3 - 9x^2 + x + 12$.

Sol. To prove that (x + 1) and (2x - 3) are factors of $2x^3 - 9x^2 + x + 12$ it is sufficient to show that p(-1) and $p\left(\frac{3}{2}\right)$ both are equal to zero.

$p(-1) = 2(-1)^3 - 9(-1)^2 + (-1) + 12 = -2 - 9 - 1 + 12 = -12 + 12 = 0$

And, $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right) + 12$

$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12 = \frac{27 - 81 + 6 + 48}{4} = \frac{-81 + 81}{4} = 0$

Hence, (x + 1) and (2x - 3) are the factors $2x^3 - 9x^2 + x + 12$. **Ans.**

Ex.7 Find $\frac{a}{}$ and β if x + 1 and x + 2 are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$.

Sol. When we put x + 1 = 0 or x = -1 and x + 2 = 0 or x = - 2 in p(x)

Then, p(-1) = 0 & p(-2) = 0

Therefore, $p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha\ (-1) + \beta = 0$

$\Rightarrow\ -1 + 3 + 2\alpha + \beta = 0 \Rightarrow \beta = -2\alpha - 2$ ....(i)

And, $p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha\ (-2) + \beta = 0$

$\Rightarrow\ -8 + 12 + 4\alpha + \beta = 0 \quad \Rightarrow \beta = -4\alpha - 4$ .....(i)

From equation (i) and (ii)

$-2\alpha - 2 = -4\alpha - 4$

$\Rightarrow\ 2\alpha = -2 \Rightarrow \alpha = -1$

Put α = -1 in equation (i) ⇒ β = -2(-1) - 2 = 2 - 2 = 0.

Hence, α = - 1 β = 0. **Ans.**

Ex.8 What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$.

Sol. Let $p(x) = 3x^3 + x^2 - 22x + 9$ and $q(x) = 3x^2 + 7x - 6$.

We know if p(x) is divided by q(x) which is quadratic polynomial therefore if p(x) is not exactly divisible by q(x) then the remainder be r(x) and degree of r(x) is less than q(x) (or Divisor)

∴ By long division method

Let we added ax + b (linear polynomial) is p(x), so that p(x) + ax + b is exactly divisible by $3x^2 + 7x - 6$.

Hence $p(X) + ax + b = s(x) = 3x^3 + x^2 - 22x + 9\ ax + b$

$= 3x^3 + x^2 - x(22 - a) + (9 + b)$

$$
\begin{array}{r|l}
 & x - 2 \\
3x^2 + 7x - 6 & 3x^3 + x^2 - x(22 - a) + 9 + b \\
 & -3x^3 + 7x^2 - 6x \\
\hline
 & -6x^2 + 6x - (22 - a)x + 9 + b \\
 & \text{or} \\
 & -6x^2 + x(-16 + a) + 9 + b \\
 & -6x^2 - 14x \quad \pm 12 \\
\hline
 & x(-2 + a) + (b - 3
\end{array}
$$

Hence, x(a – 2 + b – 3 = 0 . x + 0)

$\Rightarrow\ a - 2 = 0\ \&\ b - 3 = 0$

$\Rightarrow$ a = 2 or b = 3 **Ans.**

Hence, if we add ax + b or 2x + 3 in p(x) then it is exactly divisible by $3x^2 + 7x - 6$.

Ex.9 Using factor theorem, factories :

$p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$

Sol. $45 \Rightarrow \pm1, \pm3, \pm5, \pm9, \pm15, \pm45$

if we put x = 1 in p(x)

$p(1) = 2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) - 45$

$2 - 7 - 13 + 63 - 45 = 65 - 65 = 0$

$\therefore$ x = 1 or x - 1 is a factor of p(x).

Similarly, if we put x = 3 in p(x)

$p(3) = 2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45$

$162 - 189 - 117 + 189 - 45 = 162 - 162 = 0$

Hence, x = 3 or x - 3 = 0 is the factor of p(x).

$p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$

$\therefore$ $p(x) = 2x^3(x - 1) -5x^2(x - 1) - 18(x - 1) + 45(x - 1)$

$2x^4 - 2x^3(x - 1) - 5x^2 - 18x^2 + 18x + 45x - 54$

$\Rightarrow$ $p(x) = (x - 1)(2x^3 - 5x^2 - 18x + 45)$

$\Rightarrow$ $p(x) = (x - 1)(2x^3 - 5x^2 - 18x + 45)$

$\Rightarrow$ $p(x) = (x - 1)[2x^2(x - 3) + x(x - 3) - 15(x - 3)]$

$\Rightarrow$ $p(x) = (x - 1)[2x^3 - 6x^2 + x^2 - 3x - 15x + 45]$

$\Rightarrow$ $p(x) = (x - 1)(x - 3)(2x^2 + x - 15)$

$\Rightarrow$ $p(x) = (x - 1)(x - 3)(2x^2 + 6x - 5x - 15)$

$\Rightarrow$ $p(x) = (x - 1)(x - 3)[2x(x + 3) - 5(x + 3)]$

$\Rightarrow$ $p(x) = (x - 1)(x - 3)(x + 3)(2x - 5)$

FACTORISATION OFA QUADRATIC POLYNOMIAL

For factorisation of a quadratic expression $ax^2 + bx + a$ where $a \neq 0$, there are two method.

(a) By Method of Completion of Square :

In the form $ax^2 + bx + c$ where $a \neq 0$, firstly we take 'a' common in the whole expression then factorise by converting the expression $a\left\{x^2 + \frac{b}{a}x + \frac{c}{a}\right\}$ as the difference of two squares.

Ex.10 Factorise $x^2 - 31x + 220$.

Sol. $x^2 - 31a + 220$

$= x^2 - 2.\frac{31}{2}.x + \left(\frac{31}{2}\right)^2 - \left(\frac{31}{2}\right)^2 + 220$

$= \left(x - \frac{31}{2}\right)^2 - \frac{961}{4} + 220 = \left(x - \frac{31}{2}\right)^2 - \frac{81}{4}$

$= \left(x - \frac{31}{2}\right) - \left(\frac{9}{2}\right)^2 = \left(x - \frac{31}{2} + \frac{9}{2}\right)\left(x - \frac{31}{2} - \frac{9}{2}\right)$

$= (x - 11)(x - 20)$ **Ans.**

Ex.11 Factorise : $- 10x^2 + 31x - 24$

Sol. $-10x^2 + 31x - 24$

$$= -[10x^2 - 31x + 24] = - 10\left[x^2 - \frac{31}{10}\times + \frac{24}{10}\right]$$

$$= -10\left[x^2 - 2.\frac{31}{20}.x + \left(\frac{31}{20}\right)^2 - \left(\frac{31}{20}\right)^2 + \frac{24}{10}\right]$$

$$= -10\left[\left(x - \frac{31}{20}\right)^2 - \frac{961}{400} + \frac{24}{10}\right] = -10\left[\left(x - \frac{31}{20}\right)^2 - \frac{1}{400}\right]$$

$$= -10\left[\left(x - \frac{31}{20}\right)^2 - \left(\frac{1}{20}\right)^2\right] = -10\left[x - \frac{31}{20} + \frac{1}{20}\right]\left[x - \frac{31}{20} - \frac{1}{20}\right]$$

$$= -10\left(\frac{2x-3}{2}\right)\left(\frac{5x-8}{5}\right) = -(2x-3)(5x-8) = (3-2x)(5x-8)$$ **Ans.**

(b) By Splitting the Middle Term :

In the quadratic expression $ax^2 + bx + c$, where a is the coefficient of x^2, b is the coefficient of x and c is the constant term. In the quadratic expression of the form $x^2 + bx + c$, a = 1 is the multiple of x^2 and another terms are the same as above.

There are four types of quadratic expression :

(i) $ax^2 + bx + c$ (ii) $ax^2 - bx + c$

(iii) $ax^2 - bx - c$ (iv) $ax^2 + bx - c$

Ex.12 Factorise : $2x^2 + 12\sqrt{2x} + 35$.

Sol. $2x^2 + 12\sqrt{2x} + 35$

Product ac = 70 & b = $12\sqrt{2}$

$\therefore$ Split the middle term as $7\sqrt{2}$ & $5\sqrt{2}$

$\Rightarrow$ $2x^2 + 12\sqrt{2}x + 35 = 2x^2 + 7\sqrt{2}x + 5\sqrt{2}x + 35$

$= \sqrt{2}x\left[\sqrt{2}x + 7\right] + 5\left[\sqrt{2}x + 7\right]$

$= \left[\sqrt{2}x + 5\right]\left[\sqrt{2}x + 7\right]$ **Ans.**

Ex.13 Factorise : $x^2 - 14x + 24$.

Sol. Product ac = 24 & b = -14

$\therefore$ Split the middle term as - 12 & - 2

$\Rightarrow$ $x^2 - 14x + 24 = x^2 - 12 - 2x + 24$

$\Rightarrow$ $x(x - 12) - 2(x - 12) = (x - 12)(x - 2)$ **Ans.**

Ex.14 Factorise : $x^2 - \frac{13}{24}x - \frac{1}{12}$.

Sol. $x^2 - \frac{13}{24}x - \frac{1}{12} = \frac{1}{24}[24x^2 - 13x - 2]$

Product ac = - 48 & b = - 13 $\therefore$ We split the middle term as - 16x + 3x.

$= \frac{1}{24}[24x^2 - 16x + 3x - 2]$

$= \frac{1}{24}[8x(3x - 2) + 1(3x - 2)]$

$= \frac{1}{24}(3x - 2)(8x + 1)$ **Ans.**

Ex.15 Factorise : $\frac{3}{2}x^2 - 8x - \frac{35}{2}$

Sol. $\frac{3}{2}x^2 - 8x - \frac{35}{2} = \frac{1}{2}(3x^2 - 16x - 35) = \frac{1}{2}(3x^2 - 21x + 5x - 35)$

$= \frac{1}{2}[3x(x - 7) + 5x(x - 7)] = \frac{1}{2}(x - 7)(3x + 5)$ **Ans.**

(c) Integral Root Theorem :

If f(x) is a polynomial with integral coefficient and the leading coefficient is 1, then any integer root of f(x) is a factor of the constant term. Thus if $f(x) = x^3 - 6x^2 + 11x - 6$ has an Integral root, then it is one of the factors of 6 which are $\pm 1, \pm 2, \pm 3, \pm 6$..

Now Infect $f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

$f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$

$f(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$

Therefore Integral roots of f(X) are 1,2,3.

(d) Rational Root Theorem :

Let $\frac{b}{c}$ be a rational fraction in lowest terms. If $\frac{b}{c}$ is a root of the polynomial f(x) = $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0, a_n \neq 0$ with integral coefficients. Then b is a factor of constant term a_0 and c is a factor of the leading coefficient a_n.

For example : If $\frac{b}{c}$ is a rational root of the polynomial $f(x) = 6x^3 + 5x^2 - 3x - 2$, then the values of b are limited to the factors of -2 which are $\pm 1, \pm 2$ and the value of c are limited to the factors of 6, which are $\pm 1, \pm 2, \pm 3, \pm 6$ Hence, the possible rational roots of f(x) are $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{2}{3}$. $\pm 1, \pm 2, \pm 3, \pm 6$. Infect -1 is a Integral root and $\frac{2}{3}, -\frac{1}{2}$ are the rational roots of $f(x) = 6x^3 + 5x^2 - 3x - 2$.

NOTE:

(i) An n^{th} degree polynomial can have at most n real roots.

(ii) Finding a zero or root of polynomial f(x) means solving the polynomial equation f(x) = 0. It follows from the above discussion that if f(x) = ax + b, $a \neq 0$ is a linear polynomial, then it has only one root given by f(x) = 0 i.e. f(x) = ax + b = 0

$\Rightarrow$ ax = - b

$\Rightarrow$ $x = -\frac{b}{a}$

Thus $a = -\frac{b}{a}$ is the only root of f(x) = ax + b.

Ex.16 If $f(x) = 2x^3 - 13x^2 + 17x + 12$ then find out the value of f(-2) & f(3).

Sol. $f(x) = 2x^3 - 13x^2 + 17x + 12$

$f(-2) = 2(-2)^3 - 13(-2)^2 + 17(-2) + 12$

$= -16 - 52 - 34 + 12 = -90$ **Ans.**

$f(3) = 2(3)^3 - 13(3)^2 + 17(3) + 12$

$= 54 - 117 + 51 + 12 = 0$ **Ans.**

(e) Factorisation of an Expression Reducible to A Quadratic Expression :

Ex.17 Factorise :- $8 + 9(a - b)^6 - (a - b)^{12}$

Sol. $-8 + 9(a - b)^6 - (a - b)^{12}$

Let $(a - b)^6 = x$

Then $-8 + 9x - x^2 = -(x^2 - 9x + 8) = -(x^2 - 8x - x + 8)$

$= -(x - 8)(x - 1)$

$= -[(a - b)^6 - 8][(a - b)^6 - 1]$

$= [1 - (a - b)^6][(a - b)^6 - 8]$

$= [(1)^3 - \{(a - b)^2\}^3][\{(a - b)^2\}^3 - (2)^3]$

$= [1 - (a - b)^2][1 + (a - b)^4 + (a - b)^2][(a - b)^2 - 2][(a - b)^4 + 4 + 2(a - b)^2]$ **Ans.**

Ex.18 Factorise : $6x^2 - 5xy - 4y^2 + x + 17y - 15$

Sol. $6x^2 + x[1 - 5y] - [4y^2 - 17y + 15]$

$= 6x^2 + x[1 - 5y] - [4y^2 - 17y + 15]$

$= 6x^2 + x[1 - 5y] - [4y(y - 3) -5(y - 3)]$

$= 6x^2 + x[1 - 5y] - (4y - 5)(y - 3)$

$= 6x^2 + 3(y - 3)x - 2(4y - 5)x - (4y -)(y - 3)$

$= 3x[2x + y \;\; 3] - (4y - 5)(2x + y - 3)$

$= (2x + y - 3)(3x - 4y + 5)$ **Ans.**

EXERCISE

OBJECTIVE DPP - 5.1

1. Factors of $(42 - x - x^2)$ are:

(A) $(x - 7)(x - 6)$ (B) $(x + 7)(x - 6)$ (C) $(x + 7)(6 - x)$ (D) $(x + 7)(x + 6)$

2. Factors of $\left(x^2 + \frac{x}{6} - \frac{1}{6}\right)$ are :

(A) $\frac{1}{6}(2x + 1)(3x + 1)$ (B) $\frac{1}{6}(2x + 1)(3x - 1)$ (C) $\frac{1}{6}(2x - 1)(3x - 1)$ (D) $\frac{1}{6}(2x - 1)(3x + 1)$

3. Factors of polynomial $x^3 - 3x^2 - 10x + 2x$ are :

(A) $(x - 2)(x + 3)(x - 4)$ (B) $(x + 2)(x + 3)(x + 4)$

(C) $(x + 2)(x - 3)(x - 4)$ (D) $(x - 2)(x - 3)(x - 4)$

4. If $(x + a)$ is a factor of $x^2 + px + q$ and $x^2 + mx + n$ then the value of a is :

(A) $\frac{m-p}{n-q}$ (B) $\frac{n-q}{m-p}$ (C) $\frac{n+q}{m+p}$ (D) $\frac{m+p}{n+q}$

SUBJECTIVE DPP - 5.2

1. Factorise : $8x^3 + 16 - 9$.

2. Factorise : $x^4 + x^3 - 7x^2 - x + 6$.

3. Factorise : $9z^3 - 27z^2 - 100z + 300$.

4. Determine whether $x - 3$ is a factor of polynomial $p(x) = x^3 - 3x^2 + 4x - 12$.

5. Using factor theorem, prove that p(x) is divisible by g(x) if $P(x) = 4a^4 + 5x^3 - 12x^2 - 11x + 5$, $g(x) = 4x + 5$.

6. Determine if $(x + 1)$ is a factor of $x^3 - x^2 - (2 - \sqrt{2})x + \sqrt{2}$.

7. $x^3 - 23x^2 + 142x - 120$.

8. $x^3 + 13x^2 + 32x + 20$.

9. $2x^3 + y^2 - 2y - 1$.

10. $4z^3 + 20z^2 + 33z + 18$.

11. $x^4 + 5x^2 + 4$.

12. $x^3 - 10x^2 - 53x - 42$.

ANSWER KEY

(Objective DPP # 4.1)

Qus.	1	2	3	4	5	6	7	8	9	10	11
Ans.	A	C	C	D	C	D	A	B	C	C	D

(Subjective DPP # 4.2)

1. 36

2. 1

3. (i) $25x^2 + 40xy + 16y^2$ (ii) $16x^2 - 40xy + 25y^2$ (iii) $4x^2 - 4 + \frac{1}{x^2}$

4. 189

5. 364

6. (i) -281250 (ii) $-\frac{5}{12}$ (iii) -0.018

7. (i) $x^2 + 11x + 28$ (ii) $x^2 + \frac{26}{5}x + 1$ (iii) $P^4 + \frac{63}{4}P^2 - 4$

8. (i) 10812 (ii) 999964 (iii) 1224

9. $(2x^2 + 49a^2 + 14ax)(2x^2 + 49a^2 - 14ax)$

10. $(x - 1)(x + 1)(x^2 + 1)(x^2 + x + 1)(x^2 - x + 1)(x^4 - x^2 + 1)$

11. 3

12. (i) $x^6 + 7x^5 - 3^4 + 5x^2 + \sqrt{2}x + 4$ (ii) $m^7 + 4m^6 + 8m^5 - 3m^2 + 6m - 11$

13. $(x^2 + 5x + 3)(x^2 + 5x + 7)$

14. $(4a - 3b)^3$

15. $(x - y)(x + y)^3$

(Objective DPP # 5.1)

Qus.	1	2	3	4
Ans.	C	B	A	B

(Objective DPP # 5.2)

1. $(2x - 1)(4x^2 + 2x + 9)$

2. $(x + 1)(x - 1)(x + 3)(x - 2)$

3. $(3z + 10)(z - 3)(3z - 10)$

4. Yes

6. $(x + 1)$ is a factor.

7. $(x - 1)(x - 10)(x - 12)$

8. $(x + 1)(x + 2)(x + 10)$

9. $(y - 1)(y + 1)(2y + 1)$

10. $(z + 2)(2z + 3)(2z + 3)$

11. $(x - 1)(x + 1)(x - 2)(x + 2)$

12. $(x + 1)(x - 14)(x + 3)$

COORDINATE GEOMETRY

ML - 6

CO-ORDINATE SYSTEM

In two dimensional coordinate geometry, we u se generally two types of co-ordinate system.

(i) Cartesian or Rectangular co-ordinate system.

(ii) Polar co-ordinate system.

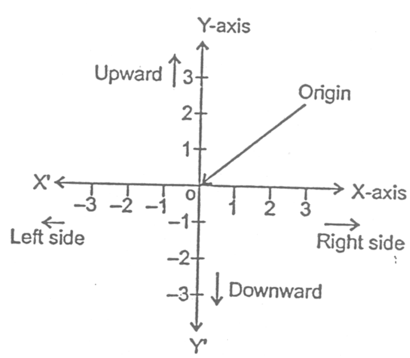
In cartesian co-ordinate system we represent any point by ordered pair (x,y) where x and y are called X and Y co-ordinate of that point respectively.

In polar co-ordinate system we represent any point by ordered pair (r, θ) where 'r' is called radius vector and $'\theta'$ is called vectorial angle of that point.

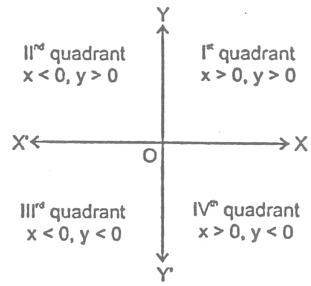
CARTESIAN CO-ORDINATE SYSTEM

(a) Rectangular Co-ordinate Axes :

Let X'OX and Y'OY are two lines such that X'OX is horizontal and Y'OY is vertical lines in the same plane and they intersect each other at O. This intersecting point is called origin. Now choose a convenient unit of length and starting from origin as zero, mark off a number scale on the horizontal line X'OX, positive to the right of origin O and negative to the left of origin O. Also mark off the same scale on the vertical line Y'OY, positive upwards and negative downwards of the origin. The line X'OX is called X-axis and the line Y'OY is known as Y-axis and the two lines taken together are called the co-ordinate axis.



(b) Quadrants :



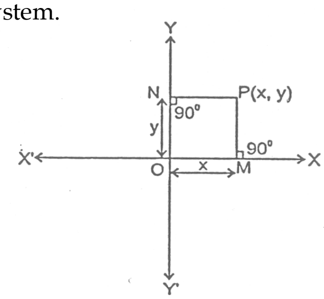
The co-ordinates axes X'OX and Y'OY divide the place of graph paper into four parts XOY, X'OY, X'OY and XOY'. These four parts are called the quadrants. The part XOY, X'OY, X'OY' and XOY' are known as the first, second, third and fourth quadrant respectively.

(c) Cartesian Co-ordinates of a Point :

Let X'OX and Y'OY be the co-ordinate axis and P be any point in the plane. To find the position of P with respect of X'OX and Y;OY, we draw two perpendiculars from P on both co-ordinate axes. Let PM and PN be the perpendiculars on X-axis and Y-axis reservedly. The length of the line segment OM is called the x-coordinate be the or abscissa of point P. Similarly the length of line segment ON is called they-coordinate or ordinate of point P.

Let OM = x and ON = y. The position of the point P in the plane with respect to the coordinate axis is represented by the ordered pair (x,y). The ordered pair (x,y) is called the coordinates of point P. "Thus, for a given point, the abscissa and ordinate are the distance of the given point from Y-axis and X-axis respectively".

The above system of coordinating on ordered pair (x,y) with every point in plane is called the Rectangular Cartesian coordinates system.



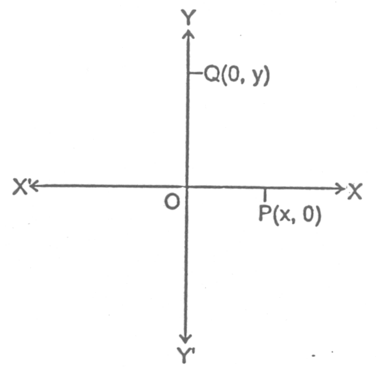
(b) Convention of Signs :

As discussed earlier that regions XOY, X'OY, X'OY' and XOY' are known as the first, second, third and fourth quadrants respectively. The ray OX is taken as positive X-axis, OX' as negative X-axis, OY as positive Y-axis and OY' as negative Y-axis. Thus we have,

In first quadrant :	$X > 0$,	$y > 0$	(Positive quadrant)
In second quadrant:	$X < 0$,	$Y > 0$	
In third quadrant :	$X < 0$,	$Y < 0$	(Negative quadrant)
In fourth quadrant:	$X > 0$,	$Y < 0$	

(e) Points on Axis :

In point P lies on X-axis then clearly its distance from X-axis will be zero, therefore we can say that its coordinate will be zero. In general, if any point lies on X-axis then its y-coordinate will be zero. Similarly if any point Q lies on Y-axis, then its distance from Y-axis will be zero therefore we can say its x-coordinate will be zero. In general, if any point lies on Y-axis then its x-coordinate will be zero.



(f) Plotting of Points :

In order to plot the points in a plane, we may use the following algorithm m.

Step I: Draw two mutually perpendicular lines on the graph paper, one horizontal and other vertical.

Step II: Mark their intersection point as O (origin).

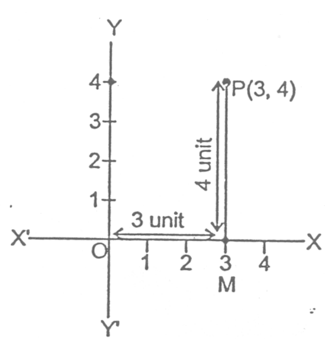
Step III: Choose a suitable scale on X-axis and Y-axis and mark the points on both the axis.

Step IV: Obtain the coordinates of the point which is to be plotted. Let the point be P(a,b). To plot this point start from the origin and |a| units move along OX, OX' according as 'a' is positive or negative respectively. Suppose we arrive at point M. From point M move vertically upward or downward |b| through units according as **'b'** is positive or negative. The point where we arrive finally is the required point P(a,b).

ILLUSTRATIONS :

Ex.1 Plot the point (3,4) on a graph paper.

Sol. let X'IX and Y'OY be the coordinate axis. Here given point is P(3,4), first we move 3 units along OX as 3 is positive then we arrive a point M. Now from M we move vertically upward as 4 is positive. Then we arrive at P(3,4).



Ex.2 Write the quadrants for the following points.

(i) A(3,4) (ii) B(-2,3) (iii) C(-5,-2) (iv) D(4,-3) (v) E(-5,-5)

Sol. (i) Here both coordinates are positive therefore point A lies in Ist quadrant.

(ii) Here x is negative and y is positive therefore point B lies in IInd quadrant.

(iii) Here both coordinates are negative therefore point C lines in IIIrd quadrant.

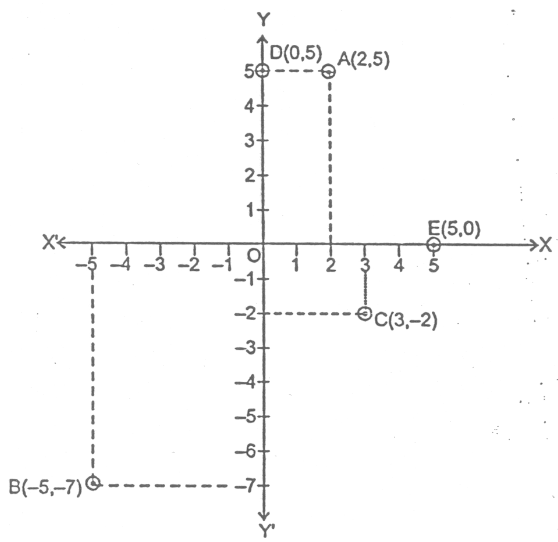
(iv) Here x is positive and y is negative therefore point D lies in IVth quadrant.

(v) Point E lies in III quadrant.

Ex.3 Plot the following points on the graph paper.

(i) A(2 ,5) (ii) B(-5,-7) (iii) C(3,-2) (iv) D (0,5) (v) E(5,0)

Sol. Let XOX' and YOY' be the coordinate axis. Then the given points may be plotted as given below :



DISTANCE BETWEEN TWO POINTS

If there are two points A (x_1, y_1) and B (x_2, y_2) on the XY plane, the distance between them is given by AB =

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ex.4 Find the distance between

(i) (5,3) and (3,2) (ii) (-1,4) and (2,-3) (iii) (a,b) and (-b,a)

Sol. Let d_1, d_2, d_3 be the required distances. By using the formula, we have

(i) $d_1 = \sqrt{(5-3)^2 + (3-2)^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$

(ii) $d_2 = \sqrt{(-1-2)^2 + \{4-(-3)\}^2} = \sqrt{(-3)^2 + 7^2} = \sqrt{58}$

(iii) $d_3 = \sqrt{\{a+(-b)\}^2 + (b-a)^2} = \sqrt{(a+b)^2 + (a-b)^2} = \sqrt{2a^2 + 2b^2}$

EXERCISE

OBJECTIVE DPP - 6.1

1. The abscissa of a point is distance of the point from :

(A) X-axis (B) Y-axis (C) Origin (D) None of these

2. The y co-ordinate of a point is distance of that point from :

(A) X-axis (B) Y-axis (C) Origin (D) None of these

3. If both co-ordinates of any point are negative then that point will lie in :

(A) First quadrant (B) Second quadrant (C) Thirst quadrant (D) Fourth quadrant

4. If the abscissa of any point is zero then that point will lie :

(A) on X-axis (B) on Y-axis (C) at origin (D) None of these

5. The co-ordinates of one end point of a diameter of a circle are (4, -1) and coordinates of the centre of the circle are (1 ,-3) then coordinates of the other end of the diameter are :

(A) (2,5) (B) (-2,-5) (C) (3,2) (D) (-3,-2)

6. The point (-2,-1), (1,0), (4,3) and (1,2) are the vertices of a :

(A) Rectangle (B) Parallelogram (C) Square (D) Rhombus

7. The distance of the point (3, 5) from X- axis is :

(A) $\sqrt{34}$ (B) 3 (C) 5 (D) None of these

SUBJECTIVE DPP - 6.2

1. Plot the points in the plane if its co-ordinates are given as A (5,0), B(0,3) C(7,2), D(-4,3), E(-3,-2) and F(3,-2).

2. In which quadrant do the following points lie A(2,3), B(-2,3), C(-3,-5), D(3, -1). Explain with reasons.

3. Plot the following pairs of numbers as points in the Cartesian plane.

x	-3	-2	8	4	0
y	5	0	3	8	-2

4. With rectangular exes, plot the points O(0,0), A(4,0) and C(0,6). Find the coordinates of the fourth points B such the OABC forms a rectangle.

5. Plot the points P(-3,1) and Q(2,1) in rectangular coordinate system and find all possible coordinates of other two vertices of a square having P and Q as two adjacent vertices".

6. Find the value of x, if the distance between the points (x, -1) and (3,2) is 5.

7. The base AB two equilateral triangles ABC and ABC′ with side 2a, lies along the x-axis such that the mid point of AB is at origin. Find the coordinates of the vertices C and C′ of the triangles.

ANSWER KEY

(Objective DPP # 6.1)

Qus.	1	2	3	4	5	6	7
Ans.	B	A	C	B	B	B	C

(Subjective DPP # 6.2)

2. A- I^{st} quadrant B - II^{nd} quadrant C III^{rd} quadrant D - IV^{th} quadrant

4. (4,6) **5.** (-3,6), (2,6) & (-3,-4), (2,-4) **6.** 7 or -1

7. $C(0, \sqrt{3}\ a), C'(0, -\sqrt{3}\ a)$

LINEAR EQUATION IN
➤➤➤ TWO VARIABLES ➤➤➤

ML - 7

LINEAR EQUATIONS IN ONE VARIABLE

An equation of the form ax + b = 0 where a and b are real numbers and 'x' is a variable, is called a linear equation in one variable.

Here 'a' is called coefficient of x and 'b' is called as a constant term. i.e. 3x + 5 = 0, 7x - 2 = 0 etc.

LINEAR EQUATION IN TWO VARIABLES

An equation of the form ax + by + c = 0 where a,b,c are real numbers and a, b $\neq$ 0, and x, y are variable, is called a linear equation in two variables, here 'a' is called coefficient of x, 'b' is called coefficient of y and 'c' is called constant term.

Any pair of values of x and y which satisfies the equation ax + by + c = 0, is called a solution of it.

Ex.1 Prove that x = 3, y = 2 is a solution of 3x - 2y = 5.

Sol. x = 3, y = 2 is a solution of 3x - 2y = 5, because L.H.S. = 3x - 2y = 3 × 3 - 2 × 2 = 9 - 4 = 5 = R.H.S.

i.e. x = 3, y = 2 satisfied the equation 3x - 2y = 5.

$\therefore$ it is solution of the given equation.

Ex.2 Prove that x = 1, y = 1 as well as x = 2, y = 5 is a solution of 4x - y - 3 = 0.

Sol. Given eq. is 4x - y - 3 = 0 ....(i)

First we put x= 1, y = 1 in L.H.S. of eq...(i)

Here L.H.S. = 4x - y - 3 = 4 × 1 - 1 - 3 = 4 - 4 = 0 = R.H.S.

Now we put x = 2, y = 5 in eq. (i)

L.H.S. = 4x - y - 3 = 4 × 2 - 5 - 3 = 8 - 8 = 0 = R.H.S.

Since, x = 1, y = 1 and x = 2, y = 5 both pair satisfied in given equation therefore they are the solution of given equation.

Ex.3 Determine whether the x = 2, y = -1 is a solution of equation 3x + 5y - 2 = 0.

Sol. Given eq, is 3x + 5y - 2 = 0 ....(i)

Taking L.H.S. = 3x + 5y - 2 = 3 × 2 + 5 × (-1) - 2 = 6 - 5 - 2 = 1 $\neq$ 0

Here L.H.S. $\neq$ R.H.S. therefore x = 2, y = - is not a solution of given equations.

GRAPH OF A LINEAR EQUATION

(A) in order to draw the graph of a linear equation in one variable we may follow the following algorithm.

Step I : Obtain the linear equation.

Step II: If the equation is of the form ax = b, a ≠ o, then plot the point $\left(\frac{b}{a},0\right)$ and one more point $\left(\frac{b}{a},\alpha\right)$ when α is any real number. If the equation is of the form ay = b, $a \neq 0$, then plot the point $\left(0,\frac{b}{a}\right)$ and $\left(\beta,\frac{b}{a}\right)$ where β is any real number.

Step III : Joint the points plotted in step II to obtain the required line.

NOTE :

If eq. is in form ax = b then we get a line parallel to Y-axis and if eg. is in form ay = b then we get a line parallel to X-axis.

Ex.4 Draw the graph of

(i) 2x + 5 = 0 (ii) 3y - 15 = 0

Sol. (i) Graph of 2x + 5 = 0

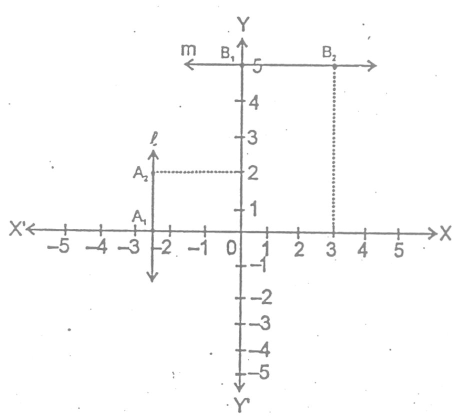
On simplifying it we get $2x = -5 \Rightarrow x = -\frac{5}{2}$

First we plot point $A_1\left(-\frac{5}{2},0\right)$ & then we plot any other point $A_2\left(-\frac{5}{2},2\right)$ on the graph paper, then we join these two points we get required line ℓ as shown in figure below.

(ii) Graph of 3y - 15 = 0

On simplifying it we get $3u = 15 \Rightarrow y = \frac{15}{3}f = 5$.

First we plot the point B_1 (0, 5) & then we plot any other point B_2 (3, 5) on the graph paper, then we join these two points we get required line m as shown in figure.



NOTE :

A point which lies on the line is a solution of that equation. A point not lying on the line is not a solution of the equation.

(B) In order to draw the graph of a linear equation ax + by + c = 0 may follow the following algorithm.

Step I : Obtain the linear equation ax + by + c = 0.

Step II : Express y in terms of x i.e. $y = -\left(\frac{ab+c}{b}\right)$ or x in terms of y i.e. $x = -\left(\frac{by+c}{a}\right)$.

Step III : Put any two or three values for x or y and calculate the corresponding values of y or x respectively from the expression obtained in Step II. Let we get points as $(\alpha_1, \beta_1), (\alpha_2, \beta_2), (\alpha_3, \beta_3)$.

Step IV : Plot the points $(\alpha_1, \beta_1), (\alpha_2, \beta_2), (\alpha_3, \beta_3)$ on graph paper.

Step V : Joint the pints marked in step IV to obtain. The line obtained is the graph of the equation ax +by + c = 0.

Ex.5 Draw the graph of the line x – 2y = 3, from the graph find the coordinate of the point when

(i) x = - 5 (ii) y = 0

Sol. Here given equation is x - 2y = 3.

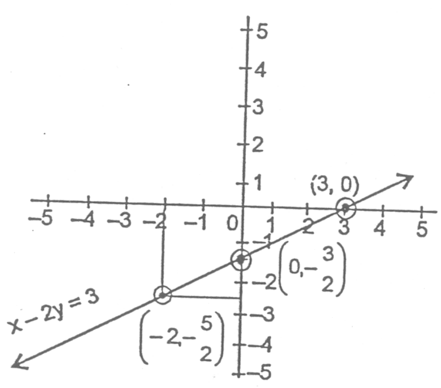
Solving it for y we get 2y = x - 3 ⇒ $y = \frac{x-3}{2}$

Let x = 0, then $y = \frac{0-3}{2} = \frac{-3}{2}$

x = 3, then $y = \frac{3-3}{2} = 0$

x = - 2, then $y = \frac{-2-3}{2} = \frac{-5}{2}$ Hence we get

x	0	3	-2
y	$-\frac{3}{2}$	0	$-\frac{5}{2}$



Clearly when x = - 5 then y = - 4 and when y = 0 then x = 3.

Ex.6 Draw the graphs of the lines represented by the equations x + y = 4 and 2x - y = 2 in the same graph. Also find the coordinate of the point where the two lines intersect.

Sol. Given equations are

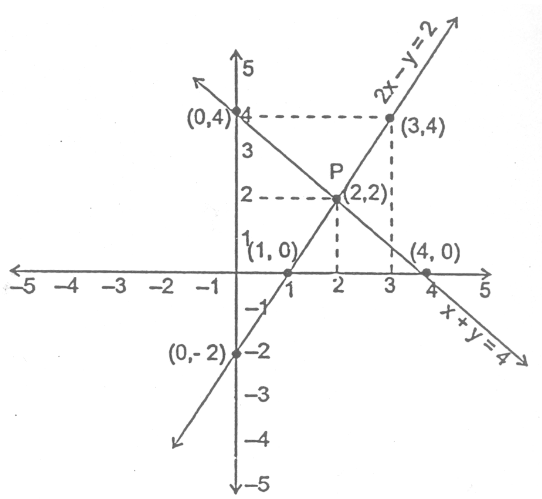
x + y = 4 ......(i) & 2x - y = 2 ......(ii)

(i) We have y = 4 - x

x	0	2	4
y	4	2	0

(ii) We have y = 2x - 2

x	1	0	3
y	0	-2	4



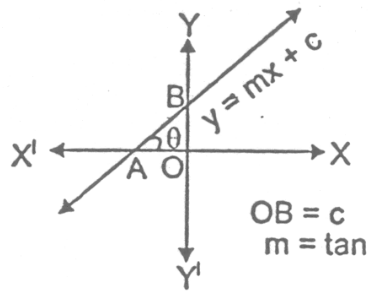
By drawing the lines on a graph paper, clearly we can say that P is the point of intersection where coordinates are x = 2, y = 2

DIFFERENCE FORMS OF A LINE

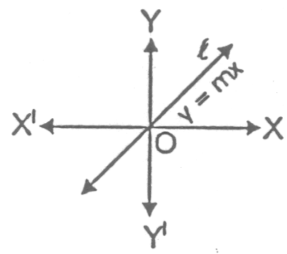
(a) Slope of a Line :

If a line makes an angle θ with positive direction of x-axis then tangent of this angle is called the slope of a line, it is denoted by m i.e. $m = \tan\theta$.

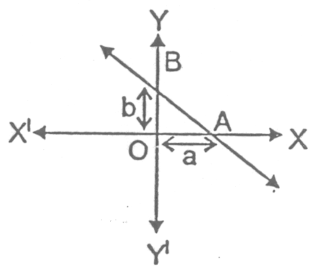
(i) Slope - intercept from is **y = mx + c** where m is the slope of line and c is intercept made by line with Y-axis.



(ii) The equation of a line passing through origin is y = mx. Here c= 0 then the line passes always from origin.



(iii) Intercept from of line is $\frac{x}{a}+\frac{y}{b}=1$ where a & b are intercepts on positive direction of x-axis and y-axis respectively made by line.



SOLUTION OF LINEAR EQUATION IN ONE VARIABLE

Let ax + b = 0 is one equation then ax + b = 0 ⇒ ax + - b ⇒ x = - $\frac{b}{a}$ is a solution.

Ex.7 Solve : $\frac{x}{2}=3+\frac{x}{3}$

Sol. Given $\frac{x}{2}=3+\frac{x}{3} \Rightarrow \frac{x}{2}-\frac{x}{3}=3$

$\Rightarrow \frac{3x-2x}{6}=3$

$\Rightarrow \frac{x}{6}=3$

$\Rightarrow x = 18$ **Ans.**

SOLUTION OF LINEAR EQUATIONS IN TWO VARIABLE

(a) By Elimination of Making Equal Coefficient :

Ex.8 Solve the following equations

2x - 3y = 5

3x + 2y = 1

Sol. Given eq. are 2x - 3y = 5 .....(i)

3x + 2y = 1 ....(ii)

Multiplying 1 eg.(i) by 3 and eg. (ii) by 2 we get

$$\begin{array}{r} 6x - 9y = 15 \\ \underline{_{-}6x +_{-} 4y = _{-}2} \\ -9y - 4y = 15 - 2 \end{array}$$

On subtraction

$\Rightarrow$ -13y = 13

$\Rightarrow$ $y = \frac{13}{-13}$

$\Rightarrow$ y = -1

Put the value of y in eg. (i) we get

2x - (3) × (-1) = 5

2x + 3 = 5

$\Rightarrow$ 2x = 5 - 3

$\Rightarrow$ 2x = 2

$\Rightarrow$ x = 1

$\therefore$ x = 1, y = 1 **Ans.**

(b) Substitution Method :

Ex.9 Solve x + 4y = 14 ....(i)

7x - 3y = ...(ii)

Sol. From equation (i) x = 14 - 4y

Substitute the value of x in equation (ii)

$\Rightarrow$ 7 (14 - 4y) - 3y = 5

$\Rightarrow$ 98 - 28y - 3y = 5

$\Rightarrow$ 98 - 31y = 5

$\Rightarrow$ 93 = 31y

$\Rightarrow$ $y = \frac{93}{31}$

$\Rightarrow$ y = 3

Now substitute value of y in equation (ii)

$\Rightarrow$ 7x - 3 (3) = 5

$\Rightarrow$ 7x - 3 (3) = 5

$\Rightarrow$ 7x = 14

$\Rightarrow$ $x = \frac{14}{7} = 2$

So, solution is x = 2 and y = 3. **Ans.**

EXERCISE

OBJECTIVE DPP # 7.1

1. Which of the following equation is not linear equation ?

(A) $2x + 3 = 7x - 2$ (B) $\frac{2}{3}x + 5 = 3x - 4$ (C) $x^2 + 3 = 5x - 3$ (D) $(x - 2)^2 = x^2 + 8$

2. Solution of equation $\sqrt{3}x - 2 = 2\sqrt{3} + 4$ is

(A) $2(\sqrt{3} - 1)$ (B) $2(1 - \sqrt{3})$ (C) $1 + \sqrt{3}$ (D) $2(1 + \sqrt{3})$

3. The value of x which satisfy $\frac{6x+5}{4x+7} = \frac{3x+5}{2x+6}$ is

(A) -1 (B) 1 (C) 2 (D) -2

4. Solution of $\frac{x-a}{b+c} + \frac{x-b}{c+a} + \frac{x-c}{a+b} = 3$ is

(A) $a + b - c$ (B) $a - b + c$ (C) $-a + b + c$ (D) $a + b + c$

5. A man is thrice as old as his son. After 14 years, the man will be twice as old as his son, then present age of this son.

(A) 42 years (B) 14 years (C) 12 years (D) 36 years

6. One forth of one third of one half of a number is 12, then number is

(A) 284 (B) 286 (C) 288 (D) 290

7. A linear equation in two variables has maximum

(A) only one solution (B) two solution (C) infinite solution (D) None of these

8. Solution of the equation $x - 2y = 2$ is/are

(A) $x = 4, y = 1$ (B) $x = 2, y = 0$ (C) $x = 6, y = 2$ (D) All of these

9. The graph of line $5x + 3y = 4$ cuts Y-axis at the point

(A) $\left(0, \frac{4}{3}\right)$ (B) $\left(0, \frac{3}{4}\right)$ (C) $\left(\frac{4}{5}, 0\right)$ (D) $\left(\frac{5}{4}, 0\right)$

10. If $x = 1, y = 1$ is a solution of equation $9ax + 12ay = 63$ then, the value of a is

(A) -3 (B) 3 (C) 7 (D) 5

SUBJECTIVE DPP - 7.2

Solve the following linear equations in one variable

1. If $\frac{2x+7}{x+2} = \frac{4x+3}{2x-7}$, find the value of $x^3 + x^2 + x + 1$.

2. Determine whether x = 5, y = 4 is a solution of the equation x - 2y = - 3

Solve the following linear equations in two variable.

3. 8x - 5y = 34, 3x - 2y = 13

4. 20x + 3y = 7, 8y - 15x = 5

5. $2x - 3y - 3 = 0,\ \frac{2x}{3} + 4y + \frac{1}{2} = 0$

6. Draw the graph of 2x + 3y = 6 and use it to find the area of triangle formed by the line and co-ordinate axis.

7. Draw the graph of the lines 4x - y = 5 and 5y - 4x = 7 on the same graph paper and find the coordinates of their point of intersection.

8. Find two numbers such that five times the greater exceeds four times the lesser by 22 and three times the greater together with seven times the lesser is 32.

9. Draw the graph of x - y + 1 = 0 and 3x + 2y - 12 = 0 on the same graph. Calculate the area bounded by these lines & X-axis.

10. If p = 3x + 1, q = $\frac{1}{3}$(9x + 13) and p : q = 6 : 5 then find x.

ANSWER KEY

(Objective DPP # 7.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	C	D	B	D	B	C	C	D	A	B

(Subjective DPP # 7.2)

1. -104

2. Yes

3. x = 3, y = -2

4. $x = \frac{1}{5}, y = 1$

5. $x = \frac{21}{20}, y = -\frac{3}{10}$

6. Area = 3 sq. units

7. x = 2, y = 3

8. 6,2

9. 7.5 sq. units

10. -7

➤➤➤ INTRODUCTION ⮜⮜⮜
TO EUCLID'S GEOMETRY

ML - 8

INTRODUCTION

The credit for introducing geometrical concepts goes to the distinguished Greek mathematician 'Euclid' who is known as the "Father of Geometry" and the word 'geometry' comes from the Geek words 'geo' which means 'Earth' and 'metreon' which means 'measure'.

BASIC CONCEPTS IN GEOMETRY

A 'point', a 'line' and a 'plane' are the basic concepts to be used in geometry.

(a) Axioms :

The basic facts which are granted without proof are called axioms.

(b) Euclid's Definitions :

(i) A point is that which has not part.

(ii) A line is breathless length.

(iii) The ends of a line segment are points.

(iv) A straight line is that which has length only.

(v) A surface is that which has length and breadth only.

(vi) The edges of surface are lines.

(vii) A plane surface is that which lies evenly with the straight lines on itself.

(c) Euclid's Five Postulates :

(i) A straight line may be drawn from any one point to any other point.

(ii) A terminated line or a line segment can be produced infinitely.

(iii) A circle can be drawn with any centre and of any radius.

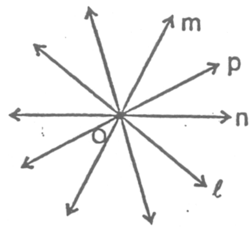
(iv) All right angles are equal to one another.

(v) If a straight line falling on two straight lines makes the exterior angles on the same side of it taken together less than two right angles, then the two straight lines if produced infinitely meet on that side on which the sum of angles are less than two right angles.

(d) Important Axioms :

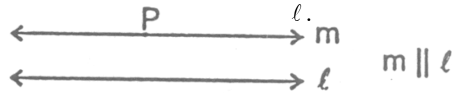
(i) A line is the collection of infinite number of points.

(ii) Through a given point, an infinite lines can be drawn.

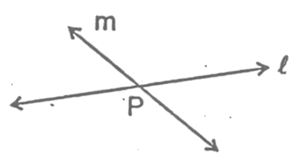


(iii) Given two distinct points, there is one and only one line that contains both the points.

(iv) If P is a point outside a line ℓ, then one and only one line can be drawn through P which is parallel to ℓ.

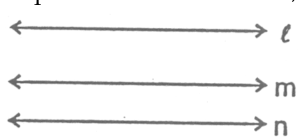


(v) Two distinct lines can not have more than one point in common.



(vi) Two lines which are both parallel to the same line, are parallel to each other.

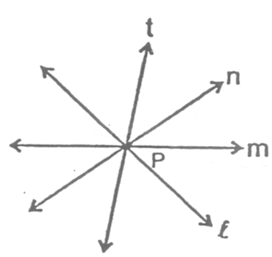
i.e. $\ell \| n, m \| n \Rightarrow \ell \| m$



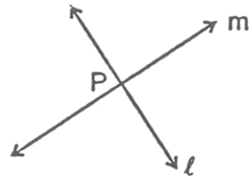
SOME IMPORTANT DEFINITOINS

(i) Collinear points : Three or more points are said to be collinear if there is a line which contains all of them.

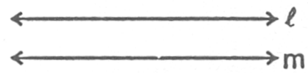
(ii) Concurrent Lines : Three or more lines are said to be concurrent if there is a point which lies on all of them.



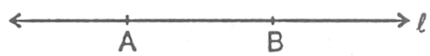
(iii) Intersecting lines : Two lines are intersecting if they have a common point. The common point is called the "point of intersection".



(iv) Parallel lines : Two lines *l* and *m* in a plane are said to be parallel lines if they do not have a common point.



(v) Line Segment : Given two points A and B on a line *l*, the connected part (segment) of the line with end points at A and B, is called the line segment AB.

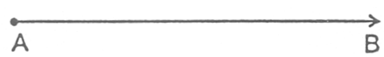


(vi) Interior point of a line segment : A point R is called an interior point of a line segment PQ if R lies between P and Q but R is neither P nor Q.

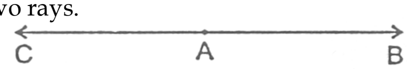
(vii) Congruence of line segment : Two line segments AB and CD are congruent if trace copy of one can be superposed on the other so as to cover it completely and exactly in this case we write $AB \cong CD$. In other words we can say two lines are congruent if their lengths is same.

(viii) Distance between two points : The distance between two points P and Q is the length of line segment PQ

(ix) Ray : Directed line segment is called a ray. If AB is a ray then it is denoted by $\overline{AB}$. Point A is called initial point of ray.



(x) Opposite rays : Two rays AB and AC are said to be opposite rays if they are collinear and point A is the only common point of the two rays.



Ex.1 If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$ AB. Explain by drawing the figure.

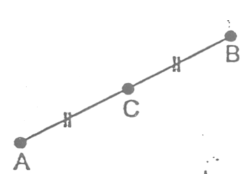
Sol. According to the given statement, the figure will be as shown alongside in which the point C lies between two points A and B such that AC = BC.

Clearly, AC + BC = AB

$\Rightarrow$ AC + AC = AB [$\because$ AC = BC]

$\Rightarrow$ 2AC = AB

And, AC = $\frac{1}{2}$AB



Ex.2 Give a definition for each of the following terms. Are there other terms that need to be defined first ? What are they, and how might you define them ?

(i) parallel lines (i) perpendicular lines (iii) line segment (iv) radius

Sol. **(i) Parallel lines :** Lines which don't intersect any where are called parallel lines.

(ii) Perpendicular lines : Two lines which are at a right angle to each other are called perpendicular lines.

(iii) Line segment : it is a terminated line.

(iv) Radius : The length of the line-segment joining the centre of a circle to any point on its circumference is called its radius.

Ex.3 How would you rewrite Euclid' fifth postulate so that it would be easier to understand ?

Sol. Two distinct intersecting lines cannot be parallel to the same line.

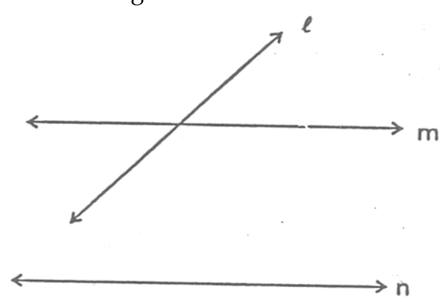
Ex.4 Does Euclid's fifth postulate imply the existence of parallel lines ? Explain.

Sol. if a straight line ℓ falls on two straight lines m and n such that sum of the interior angles on one side of ℓ is two right angles, then by Euclid's fifth postulate the line will not meet on this side of ℓ. Next, we know that the sum of the interior angles on the other side of line ℓ also be two right angles. Therefore they will not meet on the other side. So, the lines m and n never meet and are, therefore parallel.

Theorem 1 : If ℓ, m, n are lines in the same plane such that ℓ intersects m and n $\parallel$ m, then ℓ intersects n also.

Given : Three lines ℓ, m, n in the same plane s.t. ℓ intersects m and n $\parallel$ m.

To prove : Lines ℓ and n are intersecting lines.



Proof : Let ℓ and n be non intersecting lines. Then. $\ell \parallel$ n.

But,n $\parallel$ m [Given]

$\therefore$ $\ell \parallel$ n and n $\parallel$ m $\Rightarrow \ell \parallel$ m

$\Rightarrow$ ℓ and m are non-intersecting lines.

This is a contradiction to the hypothesis that ℓ and m are intersecting lines.

So our supposition is wrong.

Hence, ℓ intersects line n.

Theorem 2 : If lines AB, AC, AD and AE are parallel to a line ℓ, then points A, B, C, D and E are collinear.

Given : Lines AB, AC, AD and AE are parallel to a line ℓ.

To prove : A, B, C, D, E are collinear.

Proof : Since AB, AC, AD and AE are all parallel to a line ℓ Therefore point A is outside ℓ and lines AB, AC, AD, AE are drawn through A and each line is parallel to ℓ.

But by parallel lines axiom, one and only one line can be drawn through the point A outside it and parallel to ℓ.

This is possible only when A, B, C, D, and E all lie on the same line. Hence, A, B, C, D and E are collinear.

EXERCISE

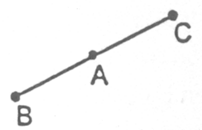
SUBJECTIVE DPP # 8.1

1. How many lines can pass through :

(i) one point (ii) two distinct points

2. Write he largest number of points in which two distinct straight lines may intersect.

3. A, B and C are three collinear points such that point A lines between B and C. Name all the line segments determined by these points and write the relation between them.



4. State, true of false :

(i) A point is a undefined term

(ii) A line is a defined term.

(iii) Two distinct lines always intersect at one point.

(iv) Two distinct point always determine a line.

(v) A ray can be extended infinitely on both the sides of its.

(vi) A line segments has both of its end-points fixed and so it has a definite length.

5. Name three undefined terms.

6. If AB is a line and P is a fixed point, outside AB, how many lines can be drawn through P which are :

(i) parallel to AB

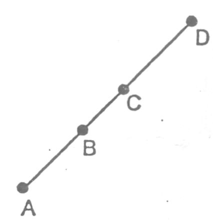
(ii) Not parallel to AB

7. Out of the three lines AB, CD and EF, if AB is parallel to EF and CD is also parallel to EF, then what is the relation between AB and CD.

8. If A, B and C three points on a line, and B lines between A and C, then prove that :

AB + BC = AC.

9. In the given figure, if AB = CD ; prove that AC = BD.



10. (i) How many lines can be drawn to pass through three given point if they are not collinear ?

(ii) How many line segments can be drawn to pass through there two given points if they are collinear

ANSWER KEY

(Subjective DPP # 8.1)

1.	(i) Infinite	(ii) Only one
2.	One	
3.	BA, AC & BC ; BA + AC = BC	
4.	(i) True	(ii) False
	(iii) False	(iv) True
	(v) False	(vi) True
5.	Point, line and plane	
6.	(i) Only one	(ii) Infinite
7.	AB ‖ CD	
10.	(i) Three lines	(ii) one

➤➤➤ LINES AND ANGLES ⮜⮜⮜

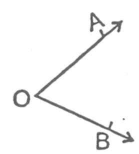
ML - 9

LINE

A line has length but no width and no thickness.

ANGLE

An angle is the union of two non-collinear rays with a common initial point. The common initial point is called the **'vertex'** of the angle and two rays are called the **'arms'** of the angles.



REMAK :

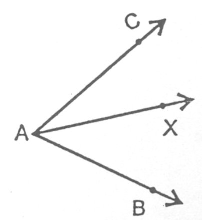
Every angle has a measure and unit of measurement is degree.

One right angle = 90^0

$1^0 = 60'$ (minutes)

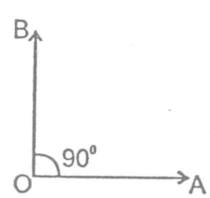
$1' = 60''$ (Seconds)

Angle addition axiom : If X is a point in the interior of ∠BAC, then m ∠BAC = m ∠BAX + m ∠XAC

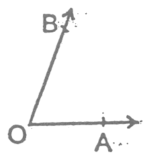


(a) Types of Angles :

(i) Right angles : An angle whose measure is 90^0 is called a right angle.

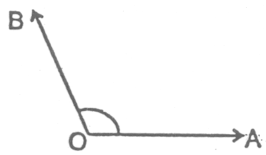


(ii) Acute angle : An angle whose measure is less than 90^0 is called an acute angle.



$0^0 < \angle BOA < 90^0$

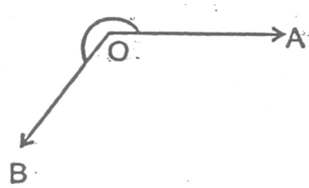
(iii) Obtuse angle : An angle whose measure is more than 90^0 but less than 180^0 is called an obtuse angle.



$90^0 < \angle AOB < 180^0$

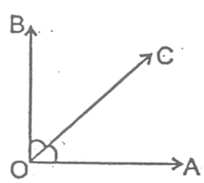
(iv) Straight angle : An angle whose measure is 180^0 is called a straight angle.

(v) Reflex angle : An angle whose measure is more than 180^0 is called a reflex angle.



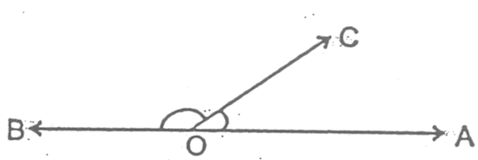
$180^0 < \angle AOB < 360^0$

(vi) Complementary angles : Two angles, the sum of whose measures is 90^0 are called complementary angles.



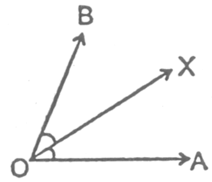
$\angle AOC$ & $\angle BOC$ are complementary as $\angle AOC + \angle BOC = 90^0$

(vii) Supplementary angles : Two angles, the sum of whose measures is 180^0, are called the supplementary angles.



$\angle AOC$ & $\angle BOC$ are supplementary as their sum is 180^0.

(viii) Angle Bisectors : A ray OX is said to be the bisector of ∠AOB, if X is a point in the interior of ∠AOB, and ∠AOX = ∠BOX.

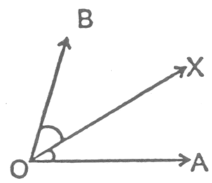


(ix) Adjacent angles : Two angles are called adjacent angles, it

(A) they have the same vertex,

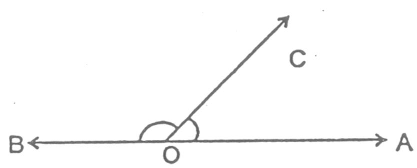
(B) they have a common arm,

(C) non common arms are on either side of the common arm.

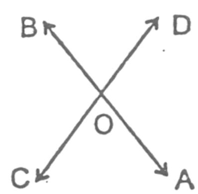


∠AOX and ∠BOX are adjacent angles, OX is common arm, OA and OB are non common arms and lies on either side of OX.

(x) Linear pair of angles : Two adjacent angles are said to form a linear pair of angles, if their non common arms are two opposite rays.



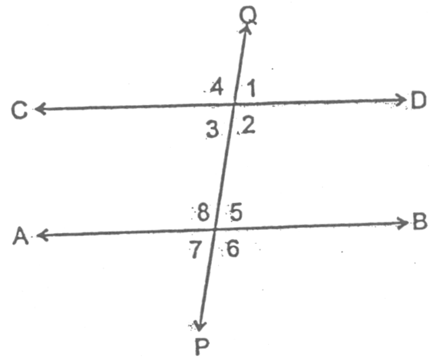
(xi) Vertically opposite angles : Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.



∠AOC & ∠BOD from a pair of vertically opposite angles. Also ∠OD & ∠BOC form a pair of vertically opposite angles.

(b) Angles Made by a Transversal with two Parallel Lines :

(i) Transversal : A line which intersects two or more give parallel lines at distinct points is called a transversal of the given lines.



(ii) Corresponding angles : Two angles on the same side of transversal are known as the corresponding angles if both lie either above the two lines or below the two lines, in figure ∠1 & ∠5, ∠4 & ∠8, ∠2 & ∠6, ∠3 & ∠7 are the pairs of corresponding angles.

(iii) Alternate interior angles : ∠3 & ∠5, ∠2 & ∠8, are the pairs of alternate interior angles.

(iv) Consecutive interior angles : The pair of interior angles on the same side of the transversal are called pairs of consecutive interior angles. In figure ∠2 & ∠5, ∠3 & ∠8, are the pair of consecutive interior angles.

(v) Corresponding angles axiom :

It a transversal intersects two parallel lines, then each pair of corresponding angles are equal. Conversely, if a transversal intersects two lines, making a pair of equal corresponding angles, then the lines are parallel.

(c) Important Facts to Remember :

(i) If a ray stands on line, then the sum of the adjacent angles so formed is 180^0.

(ii) If the sum of two adjacent angles is 180^0, then their non common arms are two apposite rays.

(iii) The sum of all the angles round a point is equal to 360^0

(iv) If two lines intersect, then the vertically opposite angles are equal.

(v) If a transversal interests two parallel lines then the corresponding angles are equal, each pair of alternate interior angles are equal and each pair of consecutive interior angles are supplementary.

(vi) if a transversal intersects two lines in such a way that a pair of alternet interior angles are equal, then the two lines are parallel.

(vii) If a transversal intersects two lines in such a way that a pair of consecutive interior angles are supplementary, then the two lines are parallel.

(viii) If two parallel lines are intersected by a transversal, the bisectors of any pair of alternate interior angles are parallel and the bisectors of an two corresponding angles are also parallel.

(ix) If a line i s perpendicular to one or two given parallel, lines, then it is also perpendicular to the other line.

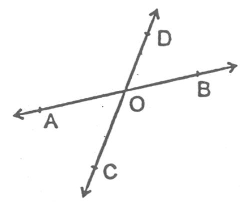
(x) Two angles which have their arms parallel are either equal or supplementary.

(xi) Two angles whose arms are perpendicular are either equal or supplementary.

IMPORTANT THEOREMS

Theorem 1 : If two lines intersect each other, then the vertically opposite angles are equal.

Given : Two lines AB and CD intersecting at a point O.



To prove : (i) $\angle AOC = \angle BOD$

(ii) $\angle BOC = \angle AOD$

Proof : Since ray OD stands on AB

$\therefore$ $\angle AOD + \angle DOB = 180^0$ ...(i) [linear pair]

again, ray OA stands on CD

$\therefore$ $\angle AOC + \angle AOD = 180^0$ ...(ii) [linear pair]

by (i) & (ii) we get

$\angle AOD + \angle DOB = \angle AOC + \angle AOD$

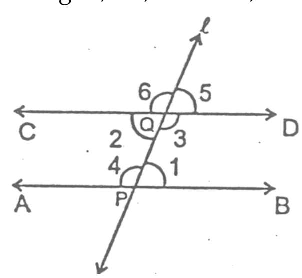
$\Rightarrow$ $\angle DOB = \angle AOC$

$\Rightarrow$ $\angle AOC = \angle DOB$

Similarly we can prove that $\angle BOC = \angle DOA$ Hence Proved.

Theorem 2 : If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

Given : AB and CD are two parallel lines, Transversal *l* intersects AB and CD at P and Q respectively making two pairs of alternate interior angles, $\angle 1$, $\angle 2$ & $\angle 3$, $\angle 4$.



To prove : $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Proof : Clearly, $\angle 2 = \angle 5$ [Vertically opposite angles]

And, $\angle 1 = \angle 5$ [Corresponding angles]

$\therefore$ $\angle 1 = \angle 2$

Also, $\angle 3 = \angle 6$ [Vertically opposite angles]

And, $\angle 4 = \angle 6$ [Corresponding angles]

$\therefore$ $\angle 3 = \angle 4$ **Hence, Proved.**

ILLUSTRATIONS

Ex.1 Two supplementary angles are in ratio 4 : 5, find the angles,

Sol. Let angles are 4x & 5x.

$\therefore$ Angles are supplementary

$\therefore$ $4x + 5x = 180^0 \Rightarrow 9x = 180^0$

$\Rightarrow$ $x = \frac{180^0}{9} = 20^0$

$\therefore$ Angles are $4 \times 20^0, 5 \times 20^0 \Rightarrow 80^0$ & 100^0 **Ans.**

Ex.2 If an angle differs from its complement by 10, find the angle.

Sol. let angles is x^0 then its complement is $90 - x^0$.

Now given $x^0 - (90 - x^0) = 10$

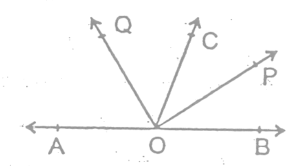
$\Rightarrow$ $x^0 - 90^0 + x^0 = 10$

$\Rightarrow$ $2x^0 = 10 + 90 = 100$

$\Rightarrow$ $x^0 = \frac{100^0}{2} = 50^0$

$\therefore$ Required angle is 50^0. **Ans.**

Ex.3 In figure, OP and OQ bisects $\angle BOC$ and $\angle AOC$ respectively. Prove that $\angle POQ = 90^0$.



Sol. $\therefore$ OP bisects $\angle BOC$

$\therefore$ $\angle POC = \frac{1}{2}\angle BOC$ ...(i)

Also OQ bisects $\angle AOC$

$\therefore$ $\angle COQ = \frac{1}{2}\angle AOC$ ...(ii)

$\therefore$ OC stands on AB

$\therefore$ $\angle AOC + \angle BOC = 180^0$ [Linear pair]

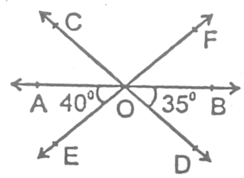
$\Rightarrow$ $\frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = \frac{1}{2} \times 180^0$

$\Rightarrow$ $\angle COQ + \angle POC = 90^0$ [Using (i) & (ii)]

$\Rightarrow$ $\angle POQ = 90^0$ [By angle sum property]

Hence Proved.

Ex.4 In figure, lines AB, CD and EF intersect at O. Find the measures of ∠AOC, ∠DOE and ∠BOF



Sol. Given ∠AOE = 40^0 & ∠BOD = 35^0

Clearly ∠AOC = ∠BOD [Vertically opposite angles]

⇒ ∠AOC = 35^0 **Ans.**

∠BOF = ∠AOE [Vertically opposite angles]

⇒ ∠BOF = 40^0 **Ans.**

Now, ∠AOB = 180^0 [Straight angles]

⇒ ∠AOC + ∠COF + ∠BOF = 180^0 [Angles sum property]

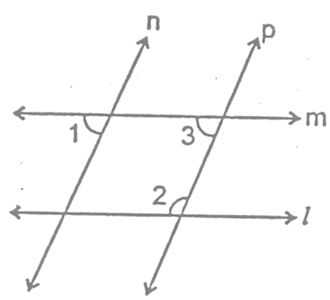
⇒ 35^0 + ∠COF + 40^0 = 180^0

⇒ ∠COF = 180^0 - 75^0 = 105^0

Now, ∠DOE = ∠COF [Vertically opposite angles]

∴ ∠DOE = 105^0 **Ans.**

Ex.5 In figure if I‖m , n‖p and ∠1 = 85^0 find ∠2



Sol. ∴ n‖p and m is transversal

∴ ∠1 = ∠3 = 85^0 [Corresponding angles]

Also m‖I & p is transversal

∴ ∠2 + ∠3 = 180^0 [∵ Consecutive interior angles]

⇒ ∠2 + 85^0 = 180^0

⇒ ∠2 + 180^0 - 85^0

⇒ ∠2 = 95^0 **Ans.**

EXERCISE

OBJECTIVE DPP # 9.1

1. If two lines intersected by a transversal, then each pair of corresponding angles so formed is "

(A) Equal (B) Complementary (C) Supplementary (D) None of these

2. Two parallel lines have :

(A) a common point (B) two common point

(C) no any common point (D) infinite common points

3. An angle is 14^0 more than its complementary angle then angle is :

(A) 38^0 (B) 52^0 (C) 50^0 (D) none of these

4. The angle between the bisectors of two adjacent supplementary angles is :

(A) acute angle (B) right angle (C) obtuse angle (D) none of these

5. If one angle of triangle is equal to the sum of the other two then triangle is :

(A) acute a triangle (B) obtuse triangle

(C) right triangle (D) none

6. X lines in the interior of $\angle BAC$. If $\angle BAC = 70^0$ and $\angle BAX = 42^0$ then $\angle XAC$ =

(A) 28^0 (B) 29^0 (C) 27^0 (D) 30^0

7. If the supplement of an angle is three times its complement, then angle is :

(A) 40^0 (B) 35^0 (C) 50^0 (D) 45^0

8. Two angles whose measures are a & b are such that $2a - 3b = 60^0$ then $\frac{4a}{5b}$ = ? If they form a linear pair :

(A) 0 (B) $\frac{8}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

9. Which one of the following statements is not false :

(A) if two angles forming a linear pair, then each of these angles is of measure 90^0

(B) angles forming a linear pair can both be acute angles

(C) one of the angles forming a linear pair can be obtuse angle

(D) bisectors of the adjacent angles form a right angle

10. Which one of the following is correct :

(A) If two parallel lines are intersected by a transversal, then alternate angles are equal

(B) If two parallel lines are intersected by a transversal then sum of the interior angles on the same side of transversal is 180^0

(C) If two parallel lines intersected by a transversal then corresponding angles are equal

(D) All of these

SUBEJCTIVE DPP # 9.2

1. The supplement of an angle is one third of itself. Determine the angle and its supplement.

2. Two complementary angles are such that two times the measure of one is equal to three times measure of the other. Find the measure of the large angle.

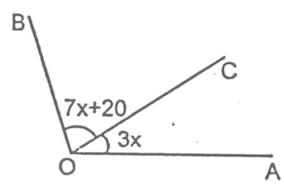
3. Find the complement of each of the following angles.

(A) $36^0 40'$ (B) $42^0 25' 36''$

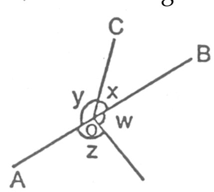
4. Write the supplementary angles of the following anglels .

(A) $54^0 28'$ (B) $98^0 35' 20''$

5. In figure, if $\angle BOC = 7x + 20^0$ and $\angle COA = 3x$, then find the value of x for which AOB becomes a straight line.

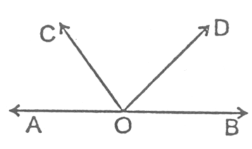


6. In figure, if x + y = w + z then prove that AOB is a straight line.

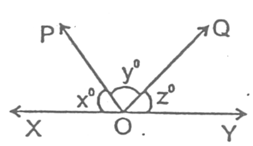


7. If the bisectors of two adjacent angles form a right angle prove that their non common angles are in the same straight line.

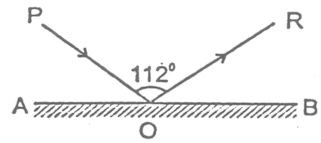
8. In figure, find $\angle COD$ when $\angle AOC + \angle BOD = 100^0$.



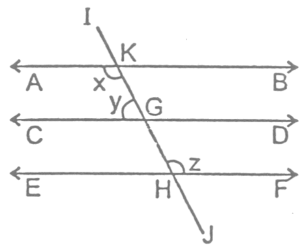
9. In figure x : y : z = 5 : 4 : 6. if XOY is a straight line find the values of x, y and z.



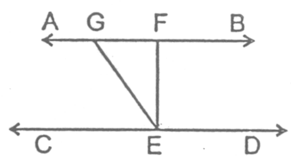
10. In the given figure, AB is a mirror, PO is the incident ray and OR, the reflected ray. If ∠POR = 112^0 find ∠POA



11. In figure, if AB ∥ CD ∥ EF and y : x = 3 : 7 find x

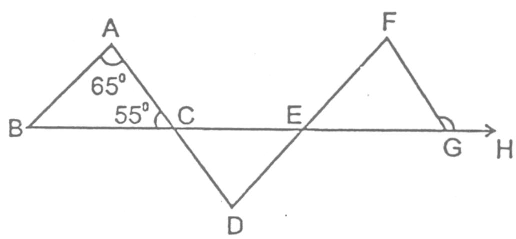


12. In figure if AB ∥ CD, EF ⊥ CD and ∠GED = 126^0, find ∠AGE, ∠GEF and ∠FGE.

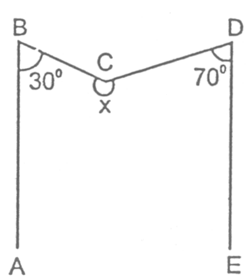


13. Δ ABC is an isosceles triangle in which ∠B = ∠V and L & M are points on AB & AC respectively such that LM ∥ BC. If ∠A = 50^0 find ∠LMC.

14. In figure if AB ∥ DF, AD ∥ FG, ∠BAC = 65^0, ∠ACB = 55^0. Find ∠FGH



15. In figure, AB ∥ ED and ∠ABC = 30^0, ∠EDC = 70^0 then find x^0.



ANSWER KEY

(Objective DPP # 9.1)

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	D	C	B	B	C	A	D	B	C	D

(Subjective DPP # 9.2)

1. 135^0, 45^0

2. 54^0

3. (A) $53^0 20'$ **(B)** $47^0 34' 24''$

4. (A) $125^0 32'$ **(B)** $81^0 24' 40''$

5. 16^0

8. 80^0

9. $60^0, 48^0, 72^0$

10. 34

11. 126^0

12. $126^0, 36^0, 54^0$

13. 115^0

14. 125^0

15. 260^0